**Acceleration and crossing of transition energy investigation using an RF structure of the Barrier Bucket type in the NICA accelerator complex.**

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INTRODUCTION

During particle acceleration to the energy of experiment in NICA collider there is a need for the crossing of transition energy . The slip-factor , which included in the equation of longitudinal motion, changes the sign, that leads to a loss of stability in the longitudinal plane when approaching to . In order to minimize the reduction of the beam parameters, a rapid jump of critical energy is assumed during the time [1] with a simultaneous change of the polarity of the RF system to ensure the stability of the beam after the jump. RF structure based on “Barrier Bucket” a feature of the NICA collider accelerating system. A non-zero value of the field between the barriers, ensures the acceleration of the beam. This feature makes the system original.

In this paper, the main features of the dynamics of the longitudinal motion of the beam crossing through the transition energy are considered, taking into account its jump in the RF structure of the “Barrier Bucket” type. Due to the rapid jump of transition energy the time at which the particles are near the zero value of the first order slip-factor is significantly reduced. Obviously, in this case, the second order of the slip-factor begins to play a decisive role in the behavior of particles inside the barrier bucket and completely determines the stability region near transition. In this case, when energy of particles crosses transition energy, focusing of the beam in the longitudinal plane disappears and the influence of the space charge becomes essential.

The change of the transition energy is carried out by change of phase of the betatron oscillations in the x-plane due to a corresponding change in the gradient of the focusing quadrupole lenses on the arcs. For such changes of the working point, it is necessary to evaluate the value of the dynamic aperture in order to determine whether it is sufficient for the stable movement of particles in the transverse plane.

RAPID JUMP OF THE TRANSITION ENERGY

When the particle energy approaches the transition energy, it is assumed to make a rapid jump in the critical energy. This can be achieved by changing the field gradient in the focusing quadrupoles of the arcs, since for periodic structures , where – first order of momentum compaction factor, – normalized frequency of betatron oscillations (tune) in the horizontal plane.The proposed fast jump in transition energy is proposed to be equal to (от ) [1]. With such a jump, the working point shifts the values from до . The maximum change rate of transition energy is limited by the parameters of quadrupoles and their power systems . Such a jump can be made during the time:

where – the time of one revolution period in the ring, – the number of revolution periods of the particles on orbit during which a transition energy jump occurs.

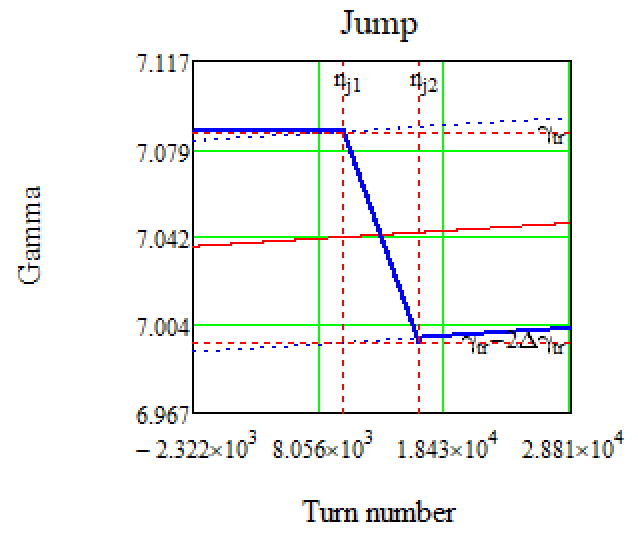
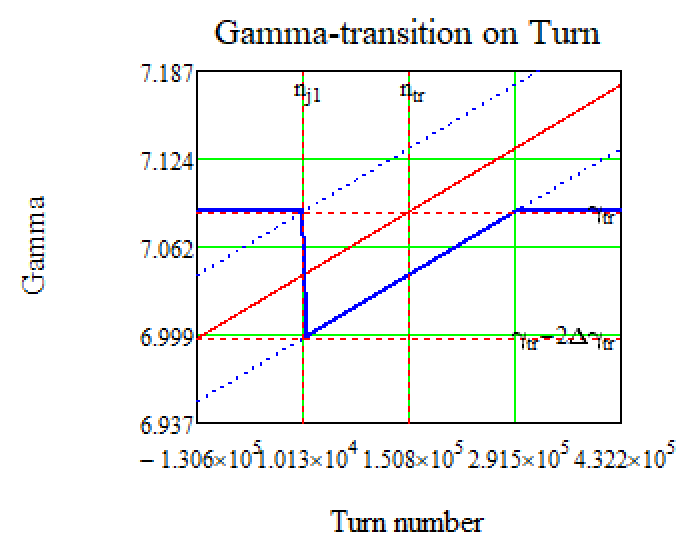
Figure 1 shows the change of transition energy together with the beam energy at the moment of the jump and after it, at a different scale.

Figure 1. On the left – a schematic diagram of the transition energy change during the jump and its recovery. On the right - the jump itself to . The blue solid line shows the transition energy , the solid red line is the energy of particles . Red hatched lines – values of and , which correspond to the moment of crossing the transition value without a jump. Blue dashed lines –  of particles shifted by .



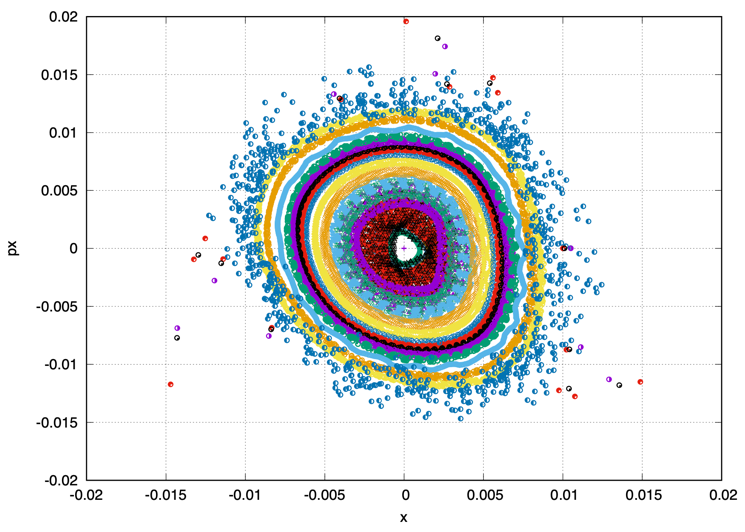
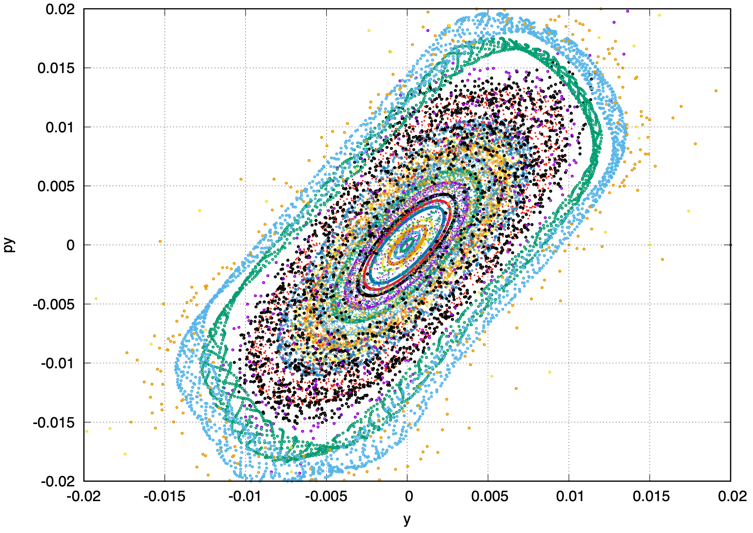
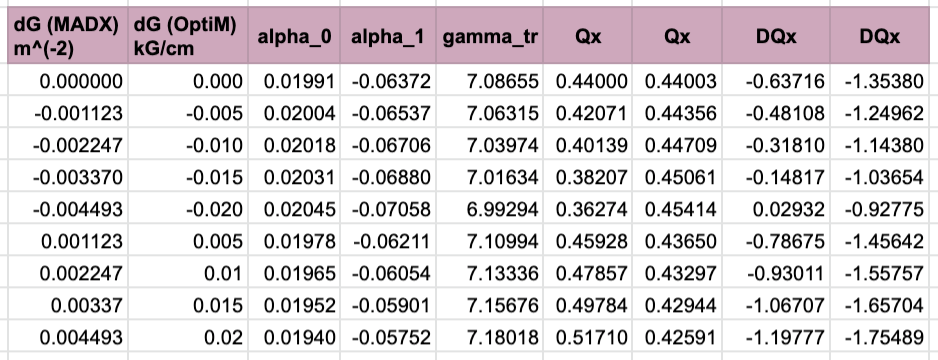
 A change of the transition energy is possible due to a change in the tune of betatron oscillations in the x-plane, which is achieved by changing the gradient of the focusing lenses in the collider arcs. With the changed parameters of quadrupole lenses, the dynamic aperture was evaluated, it determines the stable area for movement of particles in the transverse plane. The corresponding calculations were carried out using OptiM and MADX programs.

Table 1. Values depending on – variation of the gradient of focusing lenses.

Figure 2. Dynamic apertures (x–plane on the left, y–plane on the right) for the working point with with suppressed chromaticity in the collider ring, taking into account the influence of the edge sextuple components of magnets and solenoids. On the abscissa axis is the coordinate in meters, on the ordinate axis is the relative momentum in radians.

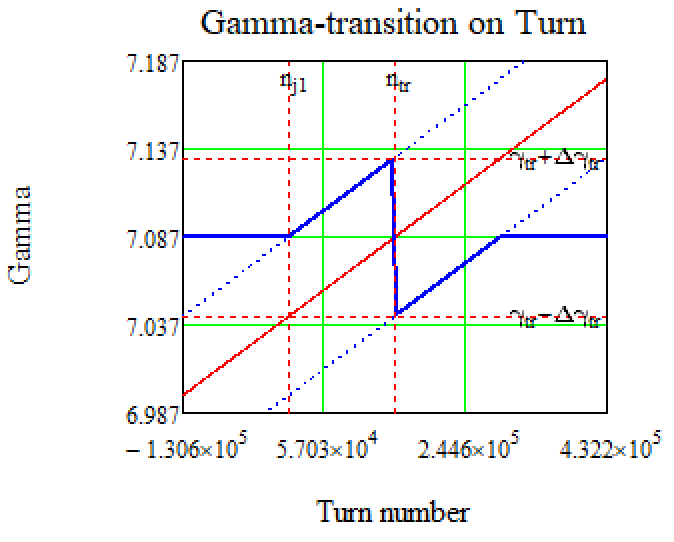
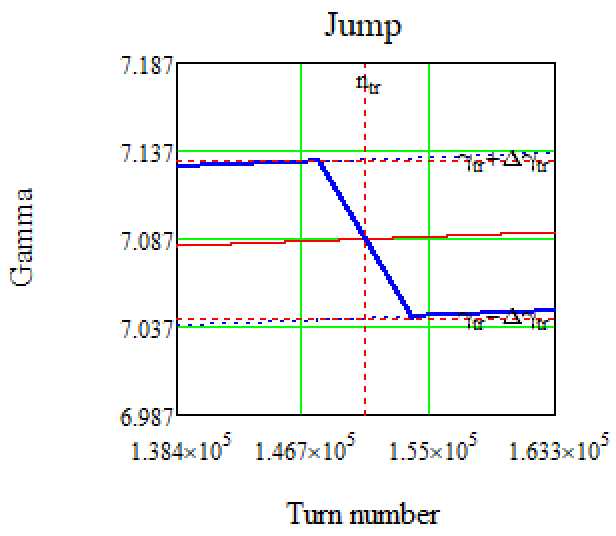
The results shown on Figure 2 indicate that the working point

it is not suboptimal, since the dynamic aperture in the vertical plane is already initially small at these operating values. Moreover, if we assume the fact that it is necessary to reach the low frequency in the horizontal plane, with and in the vertical plane (see Figure 4). So thus, the transition energy is achieved , but the dynamic aperture in the horizontal plane disappears completely at these values of betatron tunes.

Thus we considered another option (see Figure 3): first, we gradually raise the transition energy to , then we make a rapid jump down to up to . In this case, the working point changes from up to the value before the jump (see Figure 5) and after the jump down to (see Figure 6).

But in any case, we will be forced to return to the original point , which is not optimal.

Figure 3. On the left is a schematic diagram of the critical energy change , first, a smooth rise, then a quick jump and a recovery to the original value. On the right - the jump itself на . The blue solid line shows the transition energy , the red solid line is the energy of particles . Red hatched lines – values of and , which correspond to the moment of crossing the transition energy value without a jump. Blue dashed lines –  particles displaced by .



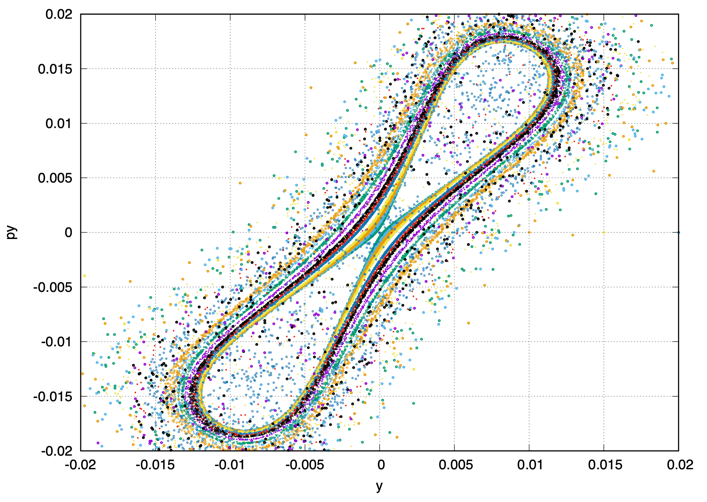
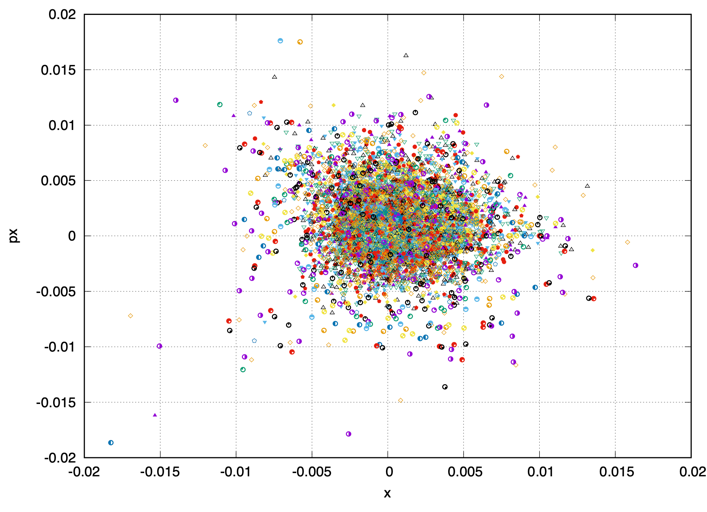
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Figure 4. Dynamic apertures (x–plane on the left, y–plane on the right) for the working point with with suppressed chromaticity in the collider ring, taking into account the influence of the edge sextuple components of magnets and solenoids. On the abscissa axis is the coordinate in meters, on the ordinate axis is the relative momentum in radians.

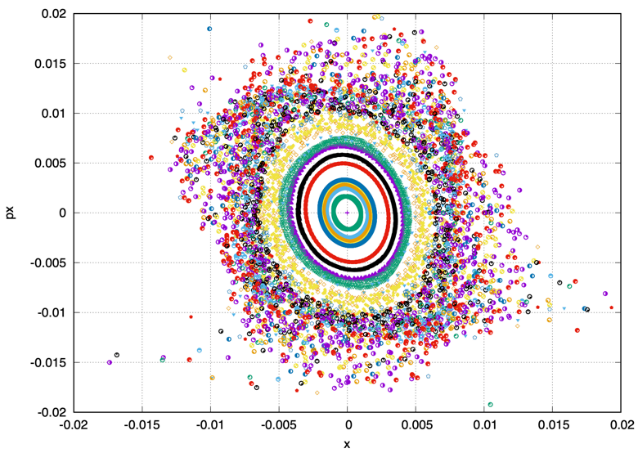
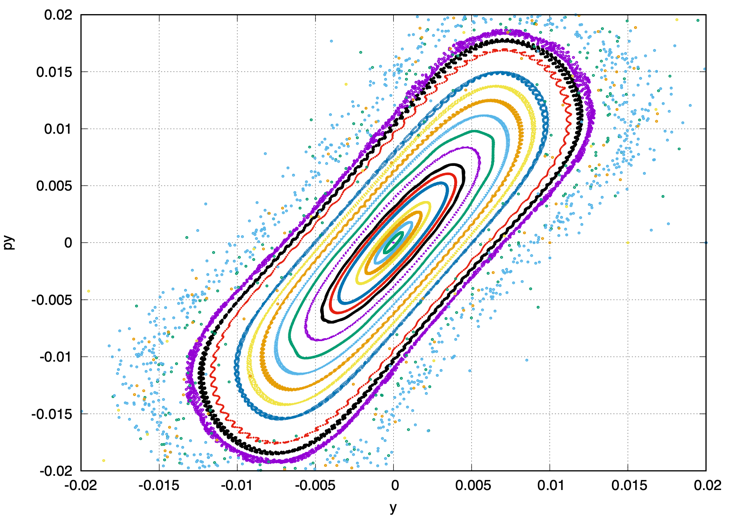
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Figure 5. Dynamic apertures (x–plane on the left, y–plane on the right) for the working point with with suppressed chromaticity in the collider ring, taking into account the influence of the edge sextuple components of magnets and solenoids. On the abscissa axis is the coordinate in meters, on the ordinate axis is the relative momentum in radians.

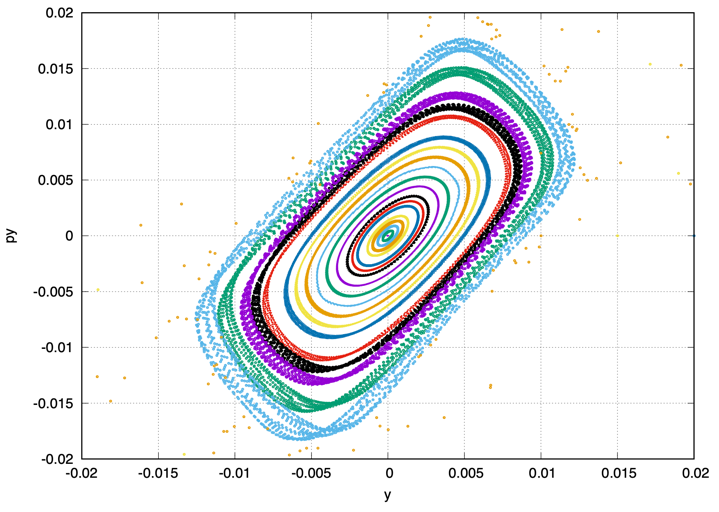
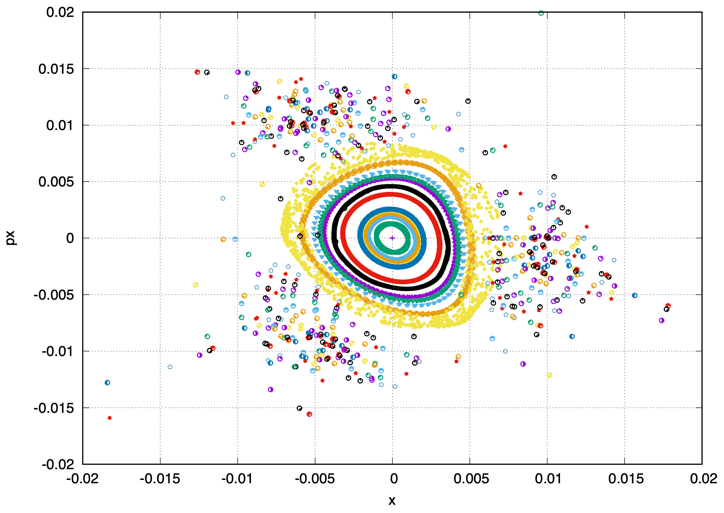
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Figure 6. Dynamic apertures (x–plane on the left, y–plane on the right) for the working point with with suppressed chromaticity in the collider ring, taking into account the influence of the edge sextuple components of magnets and solenoids. On the abscissa axis is the coordinate in meters, on the ordinate axis is the relative momentum in radians.

RF-1 SYSTEM. BARRIER BUCKET

The RF-1 system is used to retain, accumulate and accelerate particles to the experimental energy in the collider rings. Each collider ring has one RF-1 system. During retention and accumulation, 2 pairs of rectangular pulses with opposite signs are generated with the amplitude of each barrier (see Figure 7). The time duration of a single pulse can vary from . The accumulated particles enclosed between 2 pulses will be inductively accelerated by a constant potential , which is additionally created also by the RF-1 system [2].

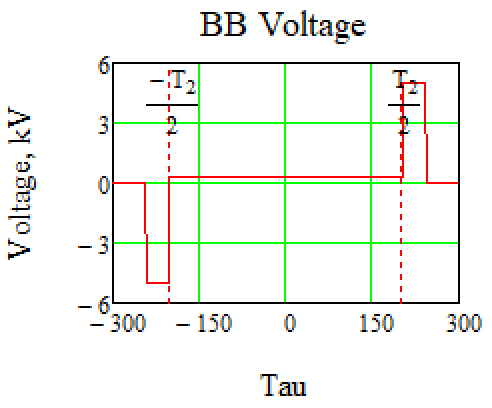
**** When the energy approaches the transition value, the RF barriers turn off and, after the proton energy becomes greater than the critical energy, the RF barriers turn on and a polarity changes. This is necessary because the slip-factor value changes it’s sign after transition energy crossing. On the one hand, when the slip-factor is purely zero the system is isochronous and with any spread of momentum, the bunch does not increase the length. On the other hand, the following second order of slip-factor begins to play an essential role η, which distorts the movement and can lead to an increase in momentum spread. And finally, in the absence of focusing in the longitudinal plane, the space charge can introduce large distortions into the phase portrait of the bunch.

Figure 7. Voltage generated by a Barrier Bucket RF station

SLIP-FACTOR NEAR TRANSITION ENERGY

During acceleration and retention of particles for away from the transition energy, the value of the slip-factor is calculated as

however, such an expression takes into account only the first order, and in general, the slip-factor depends on as [3]:

where .

Modeling the process of transition energy crossing, the first two orders of decomposition are taken into account. The corresponding momentum compaction factor (MСF) coefficients depend only on the collider magneto-optical structure.

Making a rapid jump of transition energy, parameters also undergo a jump. Value , as we have already said, it is determined by . Coefficient for different values is calculated using the program MADX/PTC [4]. Figure 8 shows the dependence of on . Table 2 shows numerical data for near transition.

Dependence of slip-factor on means that different particles having different momentum do not cross transition energy at the same time. Approaching the transition energy, the value of and begin to be comparable with each other, exactly in this area has a defining value.

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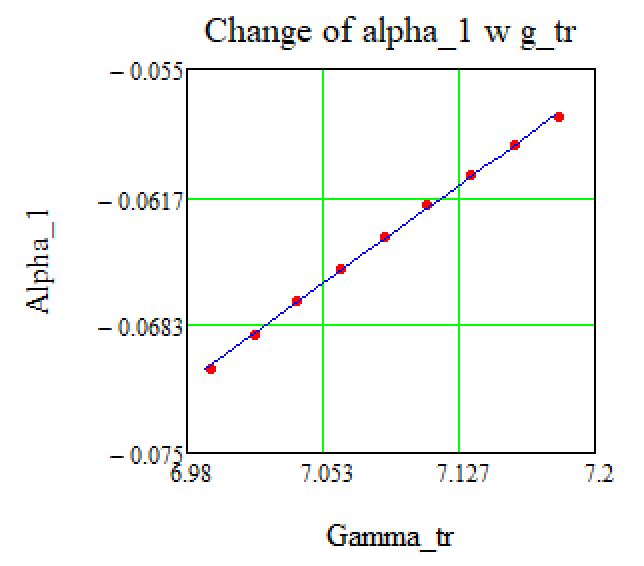


Table 2. Characteristic values of the main coefficients near transition.

Figure 8. Dependence of the coefficient on during the jump, calculated using MADX/PTC.

LONGITUDINAL DYNAMIC

Equations of longitudinal motion in Barrier Bucket in coordinates () [5,6]:

where – synchronous particle energy, – voltage generated by RF-barriers, , – harmonic number.

For modeling, in equations (4) it is convenient to switch from the time derivative to the turnover derivative : (also note that for protons )

*The influence of the second-order slip-factor on the longitudinal dynamics during the jump*

First, let's consider a jump taking into account only the first-order of the slip factor . Let's evaluate the value before the jump itself, due to the symmetry with respect to zero, it will be equal to the value after:

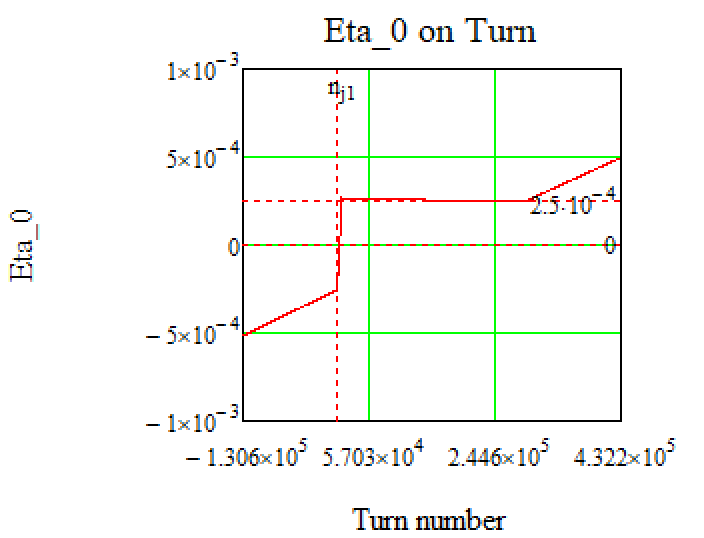
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Figure 9. Change of comes from till

Integration step by coordinate in the equations is selected in such a way that during the integration step to make a small change in the phase portrait of the beam and the size of the separatrix. Taking into account the maximum beam spread by momentum , integration step is chosen as turns.

During modeling, a random distribution is set as 2000 particles by coordinates and and the rectangle is completely filled in from till and from till (Figure 10). Energy of particles increases from to , what corresponds to the change of particles from till with .

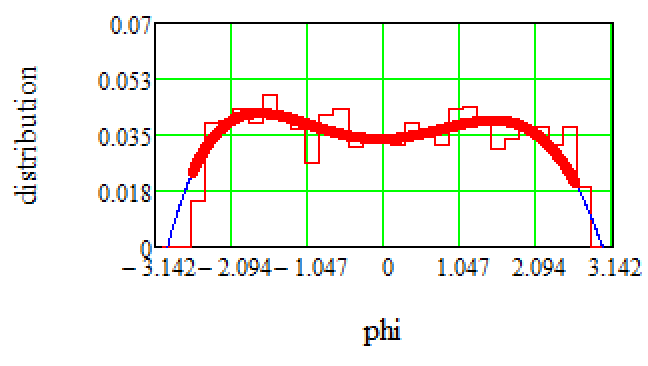
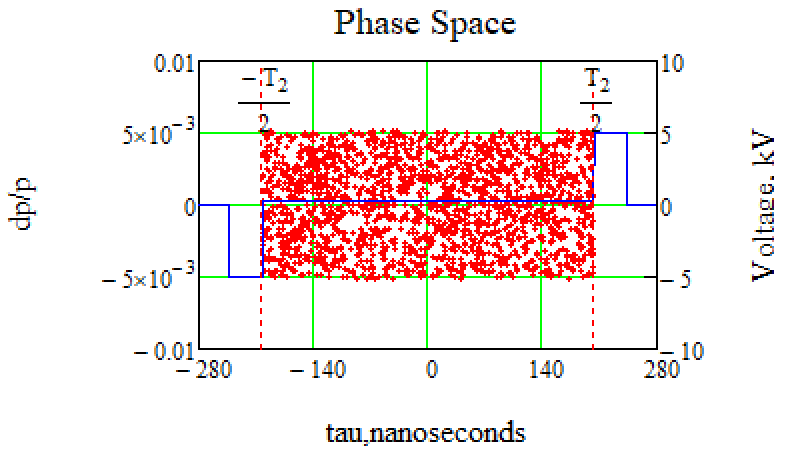
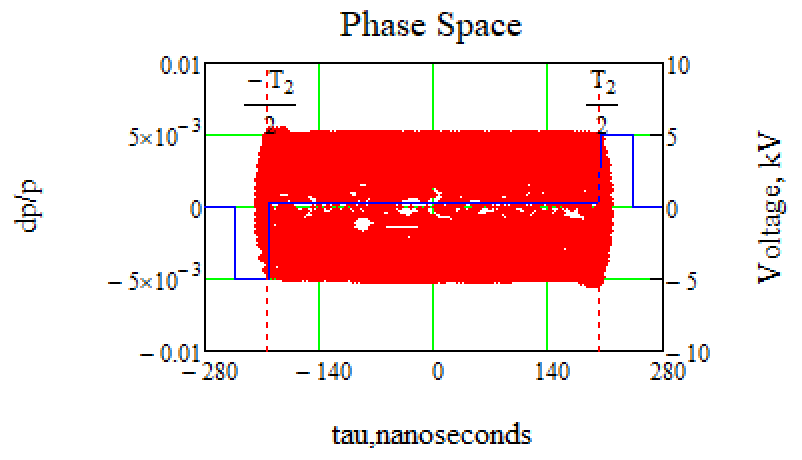
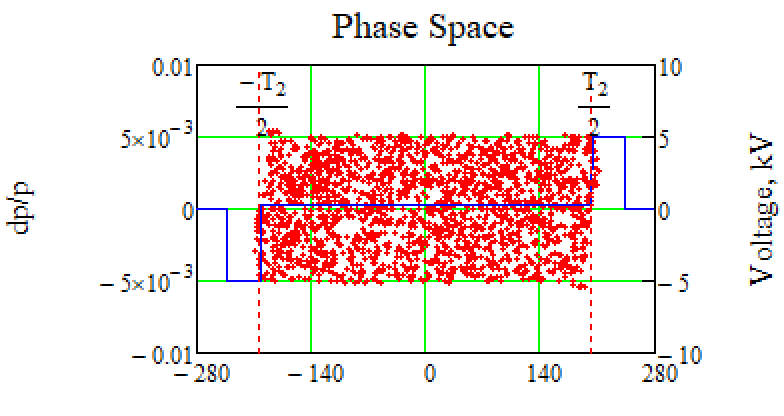


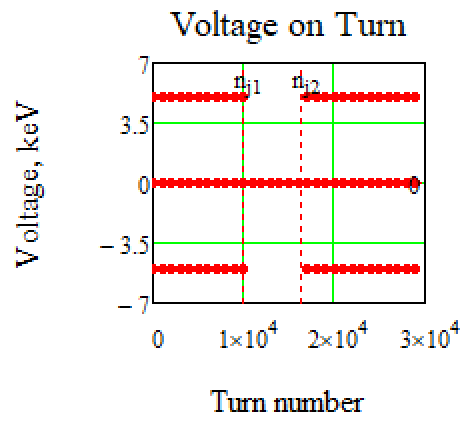
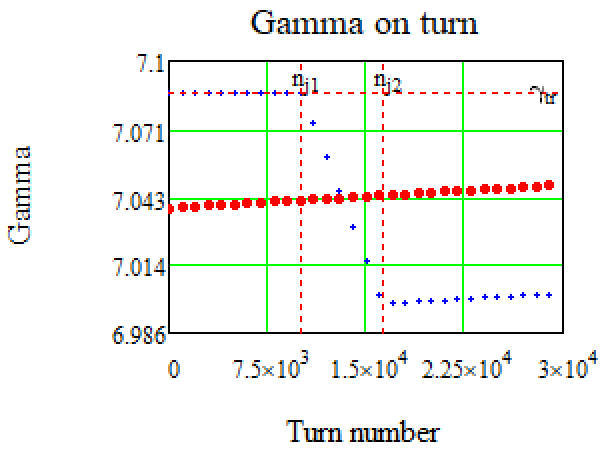
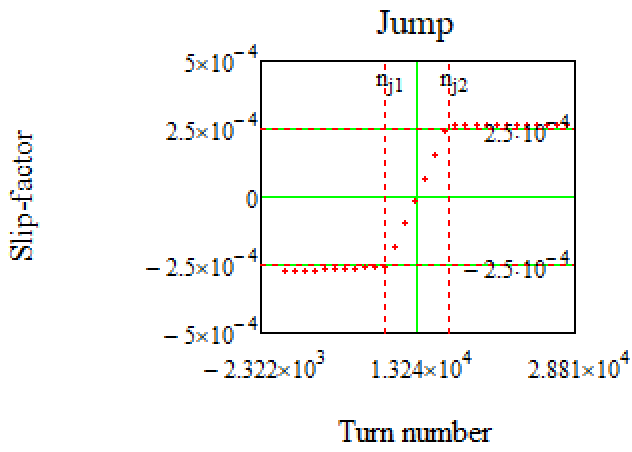
Figure 10. The initial distribution of particles inside the separatrix in coordinates is shown on the left , inside the RF barrier of the Barrier Bucket type, shown by the blue line. On the right is a histogram of the particle distribution and the corresponding approximation.

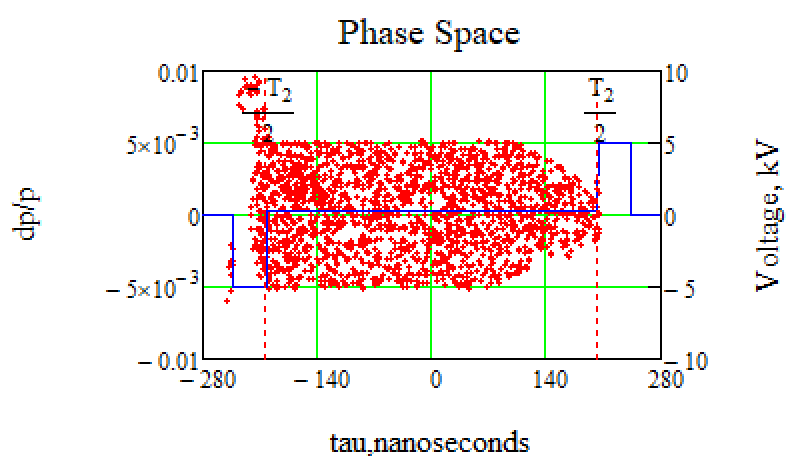
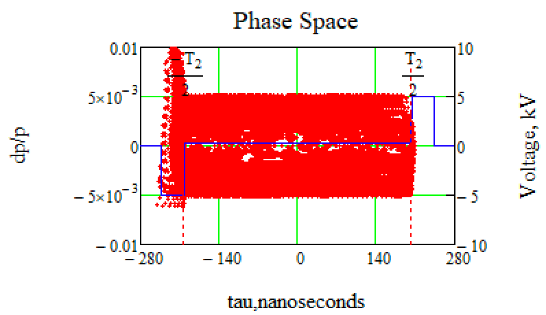
a) b)

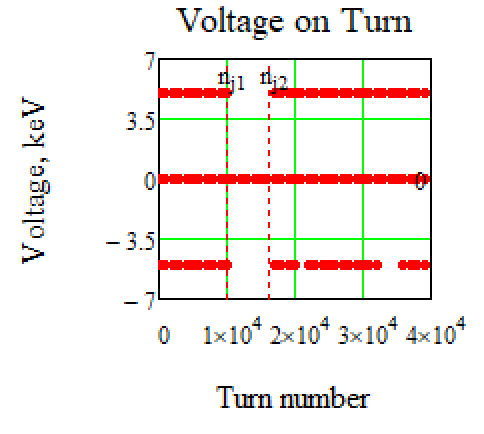
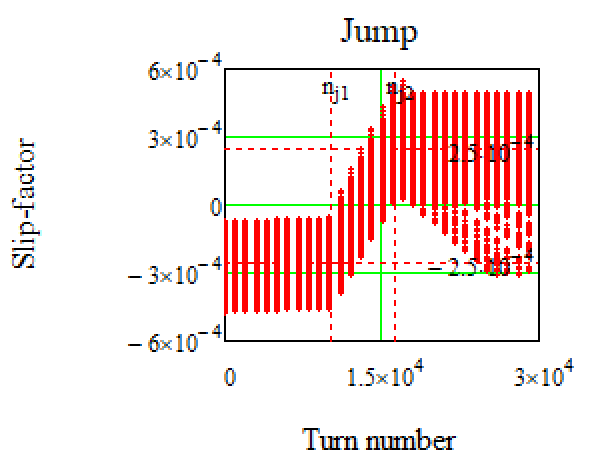
c) d) e)

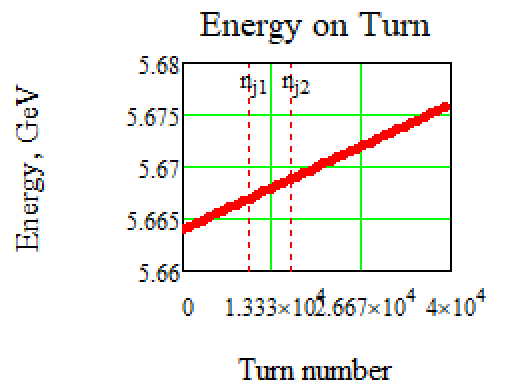
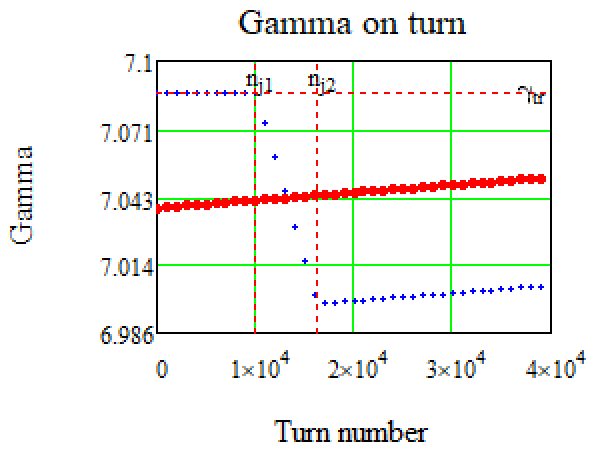
Figure 11. Crossing of the transition energy by a jump taking into account only the first order of the slip-factor

a) Blurring the phase space b) the final phase space after the jump  
c) Jump of ; d) Jump of ; e) the voltage of the RF barriers, there are no barriers when the voltage jumps.





 a) b)

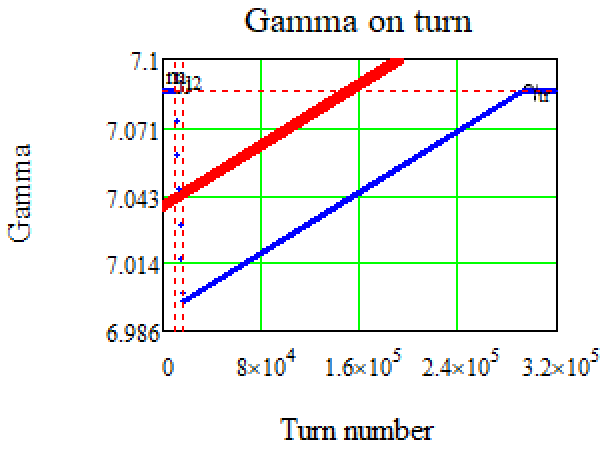
 c) d)

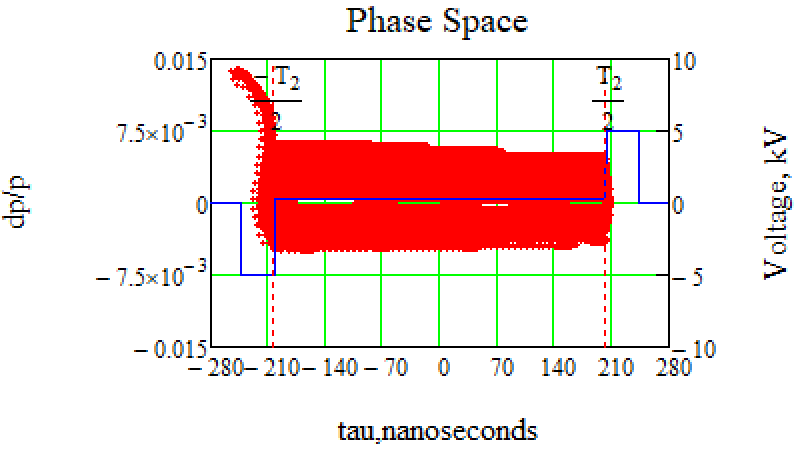
e) f)

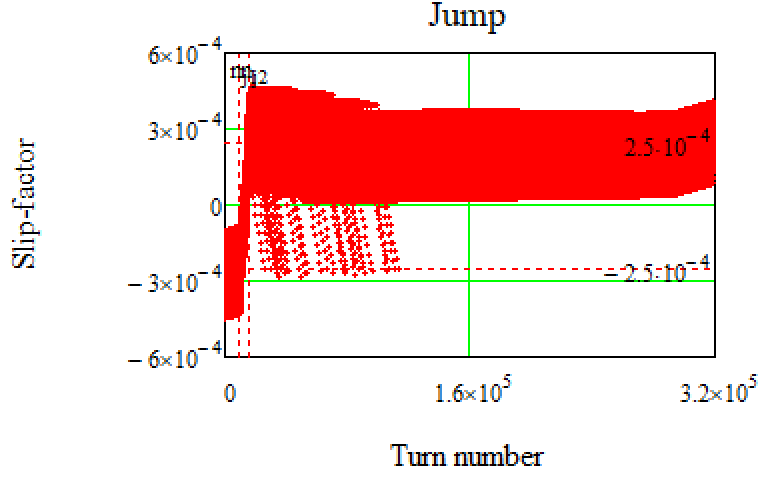
Figure 12. Crossing of the transition energy by a jump taking into account the second order of the slip-factor

a) Blurring the phase space; b) the final phase space after the jump  
c) Jump of the slip-factor (jump accurs from till ); d) the voltage of the RF barriers, there are no barriers when the voltage jumps, e) change in particle energy, f) jump of transition energy (blue dots), – gamma particles (red dots)

As it can be seen on Figure 12 (c) there is a jump of slip-factor in a different time for different particles because of dependence of slip-factor on . Obviously, after the jump, particles with a negative value of the slip-factor will not be in a stable region, since the polarity of the retaining barriers changes and will tend to leave the phase plane, as it can be seen on Figure 12 (a, b). Also, due to the momentum spread, there is an asymmetry of the phase portrait relate to the zero value of the momentum spread .

 There is still an important question: how many particles remain after the jump, when the transition energy restores to its original value before the jump. For a preliminary assessment, 200 particles were taken in the simulation instead of 2000. (Figure 13)



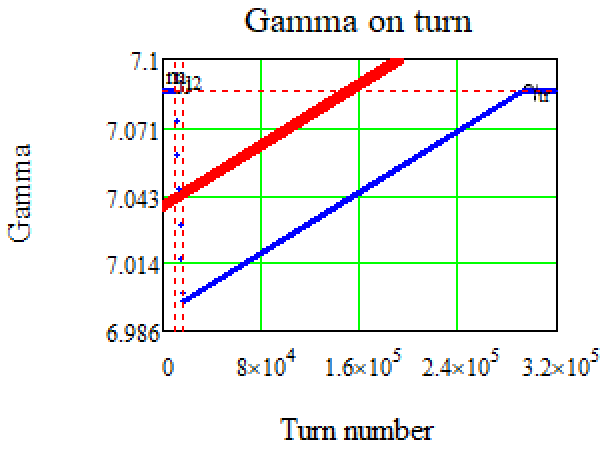
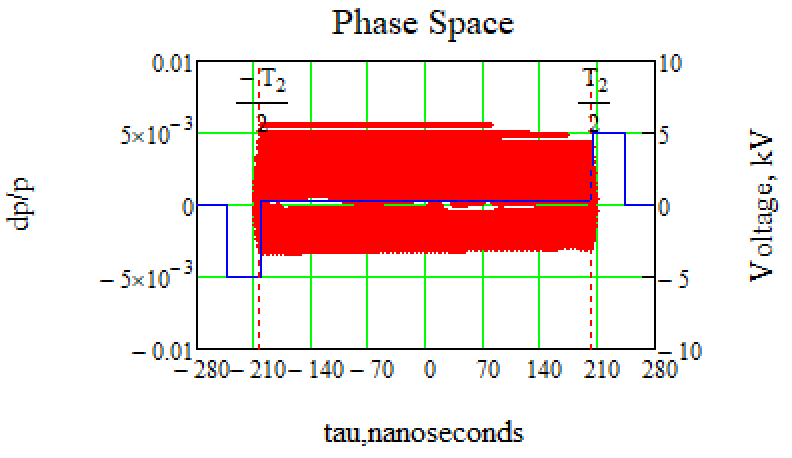
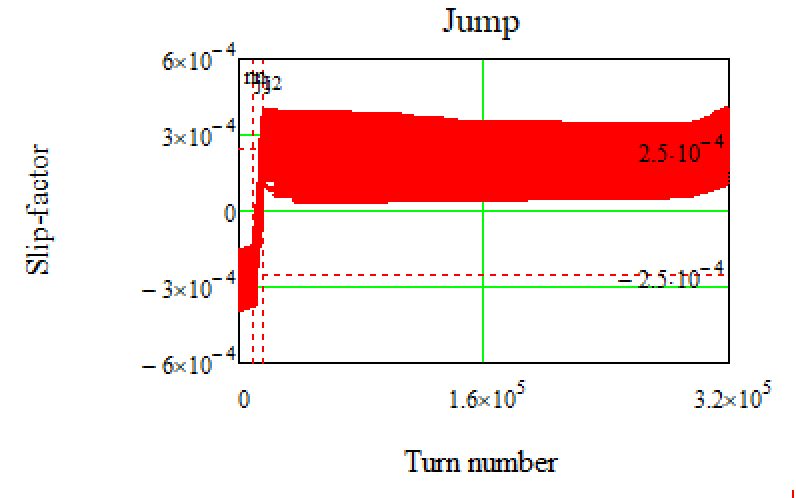
a) b)

c)

Figure 13. The output of the transition energy value to a stationary value. (initial momentum spread )

a) phase space blur; b) change of transition energy (blue dots), – gamma particles (red dots); c) changing the slip-factor .

It can be seen that after the jump itself, not all particles are retained inside the separatrix due to the change of the polarity of the RF voltage of retaining barriers. Let’s review Figure 13(a), it represents a phase space blur. Before the jump () particles with negative have a slip-factor value closer to zero than particles with positive . Because of this, the phase plane behaves asymmetrically, since the total value of the slip-factor affects the dynamics and change coordinate Thus, the particles accumulate in the area of the left barrier. During the jump, the RF is turned off and does not affect the dynamics. After the jump () the particle distribution is shifted to the left edge and now the opposite is true – for particles with positive the value of the slip-factor is closer to zero, than for particles with . Those particles that turned out to be on the left in the area of the RF barrier (with reversed polarity) due to the proximity to the zero value of the slip-factor, do not have time to stay in the separatrix. Because of this, there is a loss of particles of the order 16% for

 If we take the spread of the particle momentum as the initial one (see Figure 14), then all the particles have a time to completely change the slip-factor, the loss of particles does not occur. a) b)

c)

Figure 14. The output of the transition energy value to a stationary value. (initial momemtum spread )

a) phase space blur; b) change of transition energy (blue dots), – gamma particles (red dots); c) changing the slip-factor .

*The influence of the space charge on the longitudinal dynamics of the beam during the jump*

With this problem, it is necessary to take into account the forces of the space charge, since there is no longitudinal focusing at the moment of transition through the transition energy. The voltage generated by the space charge can be found as a derivative of the particle distribution function in the beam in -space [5]:

where – number of particles in the beam, – the number of particles in the vicinity of the coordinate ( is a distribution normalized by one), the values are calculated as , .

Thus, the equation ( taking into account (6):

During the jump, the space charge does not have a significant effect on the stability of the particles, since during the () the space charge voltage slightly changes the coordinate of particles, the characteristic voltage value is on the order of several tens of volts.

|  |  |
| --- | --- |
| harmonic number | 2 |
| radius of pipe | 5.0 sm |
| radious of beam | 0.55 sm |
| geometric factor | 5.415 |
| charge | 1 |
| constant | 377 |
| collider length | 503.04 m |
| average radius | 80 m |
| number of particles |  |

Table 3. Constants for calculating the influence of space charge.

**CONCLUSION**

In this paper, the dynamics of the longitudinal motion of particles in an RF accelerating system of the Barrier Bucket type at a transition energy jump near the transition is considered. The influence of the second order slip-factor and space charge during the transition is considered. When the critical energy jumps, the working points changes, dynamic apertures are considered for them.

For the considered jump, a change of the working point to the value is required , however, with a such change, the working point approaches a third-order resonance, which greatly affects stability, and the dynamic aperture becomes unacceptably small. However, it is possible to change the scheme of the critical energy jump and first raise the transition value, carry out a rapid jump of transition energy, and then restore the original stationary value. With this approach, the dynamic aperture remains sufficient to maintain stable motion.

During modeling and calculations, it was found that with a transition energy jump, the determining factor, which influence on the stability of the particles inside the Barrier Bucket RF system is the second order of slip-factor. It depends on momentum spread and can determine the maximum momentum spread for the transition energy crossing in a jump-scheme without loss. The space charge, due to its small size does not have time to make a significant contribution to the change in the coordinate of the particles relative to the synchronous one during jump.

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